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Theory of the Swept Intrinsic Structure

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The electric field and the hole and electron concentrations are found for reverse biased junctions in which one side is either intrinsic (I) or so weakly doped that the space charge of the carriers cannot be neglected. The analysis takes account of space charge, drift, diffusion and non linear recombination. A number of figures illustrate the penetration of the electric field into a PIN structure with increasing bias for various lengths of the I region. For the junction between a highly doped and a weakly doped region, the reverse current increases as the square root of the voltage at high voltages; and the space charge in the weakly doped region approaches a constant value that depends on the fixed charge and the intrinsic carrier concentration.

The mathematics is greatly simplified by expressing the equations in terms of the electric field and the sum of the hole and electron densities.

I. INTRODUCTION

Applications have been suggested for semiconductor structures having both extrinsic and intrinsic regions. Examples are the "swept intrinsic" structure, in which a region of high resistivity is set up by an electric field that sweeps out the mobile carriers, and the analogue transistors, where the intrinsic region is analogous to the vacuum in a vacuum tube. However, the junction between an intrinsic region and an N or P region

is considerably less well understood than the simple NP junction. Most of the assumptions that make the NP case relatively simple to deal with do not apply to junctions where one side is intrinsic. Specifically, the space charge is that of the mobile carriers; thus the flow and electrostatic problems cannot be separated as they can in PN junction under reverse bias. Tbe following sections analyze the N-intrinsic - P structure under reverse bias.

For a given material witb fairly highly doped extrinsic regions, the problem is defined by the length of the intrinsic region and the applied voltage. Taking the intrinsic region infinitely long gives the solution for a simple N-intrinsic or P-intrinsic structure. The results are given and plotted in terms of the electric field distribution. From this the potential, space charge and carrier concentrations can be found; so also can the current-voltage curve. The final section considers the case where the middle layer contains some fixed cbarge but where tbe carrier charge cannot be neglected.

Qualitative Discussion of an N-intrinsic-P Structure

Consider an N-intrinsic-P structure where the intrinsic, or I, region is considerably wider than the zero bias, or built-in, space charge regions at the junctions, so that there is normal intrinsic material between the junctions. The field distribution at zero bias can be found exactly from the zero-current analysis of Prim.[1] Tbroughout the intrinsic region, hole and electron pairs are always being tbermally generated and recombining at a rate determined by the density and properties of the traps, or recombination centers. Under zero bias the rates of generation and recombination are everywhere equal. Suppose now a reverse bias is applied causing holes to flow to the right and electrons to the left. Some of the carriers generated in the intrinsic region will be swept out before recombining. This depletes the carrier concentration in the intrinsic region and hence raises the resistivity. It also produces a space charge extending into the intrinsic layer. The electrons are displaced to the left and the holes, to the right. Tbus the space charge opposes the penetration of the field into the intrinsic region; that is, the negative charge of the electrons on the left and positive charge of the holes on the right gives a field distribution with a minimum somewhere in the interior of the intrinsic region and maxima at the NI and IP junctions. If holes and electrons had equal mobilities, the field distribution would be symmetrical with a minimum in the center of the intrinsic region. Likewise, the total carrier

[1] R. C. Prim, B. S. T. J., **32,** p. 665, May, 1953.

concentration (holes plus electrons) would be symmetrical with a maximum in the center. As the applied bias is increased the hole and electron distributions are further displaced relative to one another and the space charge increases. Finally, at high enough biases, so many of the carriers are swept out immediately after being generated that few carriers are left in the intrinsic region. Now the space charge decreases with increasing bias until there is negligible space charge, and a relatively large and constant electric field extends through the intrinsic region from junction to junction. This may happen at biases that are still much too low to appreciably affect the high fields right at the junction or in the extrinsic layers, which remain approximately as they were for zero bias.

The current will increase with voltage until the total number of carriers in the intrinsic region becomes small compared to its normal value. After that, there is negligible further increase of current with voltage. All the carriers generated in the intrinsic region are being swept out before recombining. In general, the current will saturate while the minimum field in the intrinsic region is still small compared to the average field.

Comparison with the NP Structure

The analysis is more difficult than in a simple reverse-biased *NP* structure. In the *NP* case there is a well defined space charge region in which carrier concentration is negligible compared to the fixed charge of the chemical impurities; so the field and potential distributions are easily found from the known distribution of fixed charge. Outside of the space charge region are the diffusion regions in which the minority carrier concentration rises from a low value at the edge of the space charge region to its normal value deep in the extrinsic region. However, there is no space charge in this region because the majority carrier concentration, by a very small percentage variation, can compensate for the large percentage variation in minority carrier density. The minority carriers flow by diffusion. Since the disturbance in carrier density is small compared to the majority density, the recombination follows a simple linear law (being directly proportional to the excess of minority carriers). Thus the minority carrier distribution is found by solving the simple diffusion equation with linear recombination.

None of these simplifications extend to the *NIP* or *NI* or *IP* structure. There is, in the intrinsic region, no fixed charge; hence the space charge is that of the carriers. There is no majority carrier concentration to maintain electrical neutrality outside of a limited space charge region.

It is necessary to take account of (1) space charge, (2) carrier drift, (3) carrier diffusiou and (4) recombination according to a nonlinear bimolecular law. Of these four, only space charge and recombination are never simultaneously important in practical cases. Nevertheless certain simplifications can be made if the problem is formulated so as to take advantage of them. The field and carrier distributions in the intrinsic region are found by joining two solutions: one solution is for charge neutrality; the other, which we shall call the no-recombination solution is for the case where the recombination rate is negligible compared to the rate of thermal generation of hole electron pairs. We shall show that in practical cases the ranges of validity of the two solutions overlap; that is, wherever recombination is important, we have charge neutrality.

Prim's Zero-Current Approximation

Prim* derived the field distribution in a reverse biased *NIP* structure on the assumption that the hole and electron currents are negligibly small differences between their drift and diffusion terms, as in the zero-bias case. He showed that the average diffusion current is large compared to the average current. However, as it turns out, this is misleading. Throughout almost all of the intrinsic region (where the voltage drop occurs in practical cases) the diffusion current is comparable to or smaller than the total current. The larger average diffusion current comes from the extremely large diffusion current in the small regions of high space charge at the junctions. Prim's analysis, in effect, neglects the space charge of the carriers generated in the intrinsic region. These may be neglected in calculating the field distribution if the intrinsic region is sufficiently narrow or the reverse bias sufficiently high. In the appendix we derive the limits within which Prim's calculation of the field and potential will be valid. The range will increase with both the Debye length and the diffusion length in the intrinsic material. However, in cases of practical interest the zero-current approximation may lead to serious errors in the field distribution and give a misleading idea of the penetration of the field into the intrinsic region. The present, more general analysis, reduces to Prim's near the junctions where the zero-current assumption remains valid. The zero current approximation was, of course, not intended to give the hole and electron distributions in the intrinsic region or to evaluate the effects of interacting drift, diffusion and recombination.

* Ibid.

Outline of the Following Sections

Sections II through V deal with the ideal case of equal hole and electron mobilities. Here the problem is somewhat simplified and the physics easier to visualize because of the resulting symmetry. In Section VI, the general case of arbitrary mobilities is solved by an extension of the methods developed for solving the ideal case. The technique is to deal not with the hole and electron flow densities but with two linear combinations of hole and electron flow densities that have a simple form.

Section II deals with the basic relations and in particular the formula for recombination in an intrinsic region for large disturbances in carrier density. The nature and range of validity of the various approximations are discussed. Section III derives the field distribution in regions where recombination is small compared to pair generation. Section IV treats the recombination region and the smooth joining of the recombination and no-recombination solutions. Section V considers the role of diffusion in current flow and the situation at the junctions where the field and carrier concentration abruptly become large. The change in form of the solution near the junctions is shown to be represented by a basic instability in the governing differential equation. Section VI extends the results to the general case of unequal mobilities. Section VII deals with the still more general case where there is some fixed charge in the "intrinsic" region. If the density of excess chemical impurities is small compared to the intrinsic carrier density, the solution remains unchanged in the range where recombination is important. In the no-recombination region the solution is given by a simple first order differentiatial equation which can be solved in closed form in the range where the carrier flow is by drift. The fixed charge may have a dominant effect on the space charge even when the excess density of chemical impurities is small compared to the density n_i of electrons in intrinsic material. Consider, for example, a junction between an extrinsic P region and a weakly doped n region having an excess density $N = N_d - N_a$ of donors. In the limit, as the reverse bias is increased and the space charge penetrates many diffusion lengths into the n region, the field distribution becomes linear, corresponding to a constant charge density equal to

$$\tfrac{1}{2}[N + \sqrt{N^2 + 8\, n_i^2 \mathfrak{L}^2/L_i^2}]$$

where L_i is the diffusion length in the weakly doped n type region and $\mathfrak{L}$ is the Debye length for intrinsic material. For germanium at room temperature $\mathfrak{L}/L_i$ is the order of 10^{-3}. Thus, in this example, a donor density as low as 10^{11} cm^{-3} will have an appreciable effect on the space charge.

II. BASIC RELATIONS

The problem can be stated in terms of the hole density p, the electron density n, and the electric field E and their derivatives. Let the distance x be measured in the direction from N to P. The field will be taken as positive when a hole tends to drift in the $+x$ direction. The field increases in going in the $+x$ direction when the space charge is positive. Poisson's equation for intrinsic material is

$$\frac{dE}{dx} = a(p - n) \tag{2.1}$$

where the constant a has the dimensions of volt cm and is given in terms of the electronic charge q and the dielectric constant κ by

$$a = \frac{4\pi q}{\kappa}$$

For germanium $a = 1.17 \times 10^{-7}$ volt cm.

The hole and electron flow densities J_p and J_n are[2]

$$\begin{aligned} J_p &= \mu E p - D\frac{dp}{dx} = \mu p\left[E - \frac{kT}{q}\frac{d}{dx}\ln p\right] \\ J_n &= -b\left(\mu E n + D\frac{dn}{dx}\right) = -b\mu n\left[E + \frac{kT}{q}\frac{d}{dx}\ln n\right] \end{aligned} \tag{2.2}$$

where μ and $D = \mu\, kT/q$ are the hole mobility and diffusion constant respectively, k is Boltzmann's constant (8.63×10^{-5} *ev* per °C) and T is the absolute temperature. The ratio b of electron mobility to hole mobility we take to be unity. This makes the problem symmetrical in n and p and consequently easier to understand. Section VI will extend the results to the general case of arbitrary b.

Charge and Particle Flow

For some purposes it helps to express the flow not in terms of J_p and J_n but rather in terms of the current density I and the flow density $J = J_p + J_n$ of particles, or carriers. The current density $I = q(J_p - J_n)$. Each carrier, hole or electron, gives a positive contribution to J if it goes in the $+x$ direction and a negative contribution if it goes in the $-x$ direction. In other words, J is the net flow of carriers regardless of their charge sign. The current I is constant throughout the intrinsic

[2] See, for example, Electrons and Holes in Semiconductors, by W. Shockley. D. Van Nostrand Co., New York, 1950.

region. Particle flow is away from the center of the intrinsic region. Carriers are generated in the intrinsic region and flow out at the two ends, the electrons going out on the N side and holes on the P side. Thus J is positive near the IP junction and negative near the NI junction.

From the definitions of I and J and equations (2.2)

$$\begin{aligned} \frac{I}{q} &= \mu E(p + n) - D \frac{d}{dx}(p - n) \\ J &= \mu E(p - n) - D \frac{d}{dx}(p + n) \end{aligned} \tag{2.3}$$

It is convenient to express the equations in terms of E and a dimensionless variable

$$s = \frac{n + p}{2n_i} \tag{2.4}$$

which measures how "swept" the region is. In normal intrinsic material $s = 1$. In a completely swept region $s = 0$; at the junctions with highly extrinsic material $s \gg 1$. Using Poisson's equation to express $p - n$ in terms of E, equations (2.3) become

$$\begin{aligned} I &= \sigma s E - \frac{qD}{a} \frac{d^2E}{dx^2} \\ J &= \frac{d}{dx}\left[\frac{\mu E^2}{2a} - 2n_i D s\right] \end{aligned} \tag{2.5}$$

where $\sigma = 2\,\mu\,n_i q$ is the conductivity of intrinsic material. The particle flow J is thus seen to be the gradient of a flow potential that depends only on E and s.

Equations (2.5) can be written in the form

$$I = \sigma\left[sE - \mathfrak{L}^2 \frac{d^2E}{dx^2}\right] \tag{2.6}$$

$$J = D2n_i \frac{d}{dx}\left[\frac{E^2}{E_1^2} - s\right] \tag{2.7}$$

where $\mathfrak{L} = \sqrt{kT/2an_iq}$ is the Debye length in intrinsic material and

$$E_1 = 2\sqrt{\frac{an_ikT}{q}} = \frac{\sqrt{2}kT}{q\mathfrak{L}} \tag{2.8}$$

is a field characteristic of the material and temperature. Specifically E_1 is $\sqrt{2}$ times the field required to give a voltage drop kT/q in a Debye

length. For germanium at room temperature $\mathcal{L} = 6.8\ 10^{-5}$ cm and $E_1 = 383$ volts per cm.

Both I and J are the sum of a drift term and a diffusion term. For charge neutrality, where $p - n$ is small compared to $p + n$, both charge diffusion and particle drift can be neglected. We shall see later that, except right at the junctions, charge diffusion is negligible.

The Equations of Continuity

The two equations of continuity are

$$\frac{dJ_p}{dx} = \frac{dJ_n}{dx} = g - r \tag{2.9}$$

where g is the rate of pair generation and r the rate of recombination. In terms of I and J, these become

$$\frac{dI}{dx} = 0 \tag{2.10}$$

or $I =$ constant and

$$\frac{dJ}{dx} = 2(g - r) \tag{2.11}$$

which says that the gradient of particle flow is equal to the net rate of particle generation, that is, twice the net rate of pair generation.

To complete the statement of the problem it remains to express g and r in terms of n and p.

Generation and Recombination

The direct generation and recombination of holes and electrons follows the mass action law, in which $g - r$ is proportional to $n_i^2 - np$. The constant of proportionality can be defined in terms of a lifetime τ as follows: Let $\delta p = \delta n \ll n_i$ be a small disturbance in carrier density. Then defining $\tau(g - r) = -\delta n$, we see that the proportionality constant in the mass action law is $(2n_i\tau)^{-1}$. So

$$g - r = \frac{n_i^2 - np}{2n_i\tau} \tag{2.12}$$

and the generation rate

$$g = \frac{n_i}{2\tau} \tag{2.13}$$

is independent of carrier concentration.

In actual semiconducting materials, recombination is not direct. Rather it occurs through a trap, or recombination center. The statistics of indirect recombination has been treated by Shockley and Read[3] for a recombination center having an arbitrary energy level $\mathcal{E}_t$ somewhere in the energy gap. At any temperature the trap level can be expressed by the values n_1 and p_1 which n and p would have if, at that temperature, the Fermi level were at the trap level. Shockley and Read showed that, at a given temperature, the lifetime for small disturbances in carrier density is a maximum in intrinsic material. It drops to limiting values τ_{n0} and τ_{p0} in highly extrinsic n and p material, respectively. The formula for $g - r$ in terms of n and p is

$$g - r = \frac{n_i^2 - np}{\tau_{p0}(n + n_1) + \tau_{n0}(p + p_1)} \tag{2.14}$$

For our purposes it is more convenient to define the lifetime τ not by $\tau(g - r) = -\delta n \ll n_i$, but rather as the proportionality factor in the mass action law. Then τ is not necessarily constant independent of carrier density. From (2.12) and (2.14)

$$\tau = \frac{\tau_{p0}(n + n_1) + \tau_{n0}(p + p_1)}{2n_i} \tag{2.15}$$

We shall be interested in the lifetime in the region where n and p are equal to or less than n_i. τ decreases as n and p decrease; that is, τ is less in a swept region than in normal intrinsic material. Let $\tau = \tau_i$ for $n = p = n_i$ and $\tau = \tau_0$ for $n = p = 0$. The total range of variation of τ is by a factor of

$$\frac{\tau_i}{\tau_0} = 1 + \frac{n_i(\tau_{p0} + \tau_{p0})}{p_1\tau_{n0} + n_1\tau_{p0}} \tag{2.16}$$

Let the energy levels be measured relative to the intrinsic level, and define a level $\mathcal{E}_0$ by

$$\mathcal{E}_0 = kT \ln \sqrt{\frac{\tau_{n0}}{\tau_{p0}}}$$

Then if $\mathcal{E}_t = \mathcal{E}_0$, $n_1\tau_{p0} = p_1\tau_{n0}$. Now eq. (2.16) becomes

$$\frac{\tau_i}{\tau_0} = 1 + \frac{1}{2}\left(\sqrt{\frac{\tau_{n0}}{\tau_{p0}}} + \sqrt{\frac{\tau_{p0}}{\tau_{n0}}}\right) \operatorname{sech}\left(\frac{\mathcal{E}_t - \mathcal{E}_0}{kT}\right) \tag{2.17}$$

Thus the variation in τ increases as the ratio of τ_{n0} to τ_{p0} deviates from unity and as the trap level moves away from the level $\mathcal{E}_0$.

[3] W. Shockley and W. T. Read, Jr., Phys. Rev., **87**, p. 835, 1952.

The data of Burton, Hull, Morin, and Severien[4] shows that a typical value of the ratio of τ_{p0} and τ_{n0} is about 10. This means that the variation in τ with carrier concentration will be less than 10 per cent provided $\mathcal{E}_t$ is about $4kT$ from $\mathcal{E}_0$. In what follows we shall assume that this is so. Then we have the mass action law (2.12) with τ a constant, which could be measured by one of the standard techniques involving small disturbances in carrier density. The general case of variable τ is considered briefly at the end of Section IV.

Outline of the Solution

To conclude this section, we discuss briefly the form of the equations and the solution in different parts of the intrinsic region. First consider (2.6) for the current in the ideal case of equal mobilities. In Sections III and V we shall show that throughout almost all of the intrinsic region the current flows mainly by pure drift so we can take I $= \sigma sE$. The reason for this is as follows. The quantity $\mathcal{L}^2$ is so small that the diffusion term remains negligible unless the second derivative of E becomes large — so large in fact that the E versus x curve bends sharply upward and both the drift and diffusion terms become large compared to the current I. This is the situation at the junction where I is the small difference between large drift and diffusion terms. Thus (2.6) has two limiting forms:

(1) Except at the junctions the current is almost pure drift so $I = \sigma sE$ is a good approximation. In Section III we derive an upper limit for the error introduced by this approximation and show how the approximate solution can be corrected to take account of the diffusion term.

(2) At the junction, the drift term becomes important and the current rapidly becomes a small difference between its drift and diffusion terms and the solution approaches the zero current solution, for which $sE = \mathcal{L}^2 \, d^2E/dx^2$. In Section V we derive an approximate solution that joins onto the $I = \sigma sE$ solution near the junction and then turns continously and rapidly into the zero current solution. We shall call this the *junction* solution.

The abrupt change in the solution from (1) to (2) near the junction is shown to be related to a basic instability in the differential equation. This makes it impractical to solve the equations on a machine.

When the applied bias is large compared to the built-in voltage drop, the junction region will be of relatively little interest so the $I = \sigma sE$ solution can be used throughout.

In the region where $I = \sigma sE$ there are two overlapping regions in which the equations assume a simple form. These are the following:

[4] Burton, Hull, Morin and Severiens, J. Phys. Chem., 57, p. 853, 1953.

The No-Recombination Solution

Here recombination is small compared to generation, $r \ll g$. This will be so in at least part of the intrinsic region for reverse biases of more than a few kT/q. The E versus x curve turns out to be given by a simple, cubic algebraic equation.

The Recombination, or Charge Neutrality, Solution

Here $n - p$ is small compared to $n + p$, so the particle flow is by diffusion. We shall find that the s versus x curve is given by a third degree elliptic integral. As we move away from the center of the intrinsic region and toward the junctions, recombination becomes small compared to generation and the recombination solution goes into the no-recombination solution. In the region where both solutions hold, the solution has the simple form $s = I/\sigma E = A - x^2$ where A is a constant that must be less than $\frac{2}{3}$ and the unit of length is twice the diffusion length.

As the bias on an NIP structure is increased and the space charge penetrates through the intrinsic region, the region where the recombination is important will shrink and eventually disappear.

Fig. 1 is a schematic plot of the field distribution for the case where the applied bias is large compared to the built-in potential drop but not large enough to sweep all the carriers out of the intrinsic region. As the voltage is increased, the drop in field in the intrinsic region will become less and finally the field distribution will be almost flat from junction to junction. Only half of the intrinsic region is shown in Fig. 1. For equal mobilities the field distribution will be symmetrical about the center x_i of the intrinsic region.

The illustration shows the recombination solution (1), which holds near the center of the intrinsic region and overlaps (2), the no-recombination solution. The junction solution (3) joins continuously onto the no-recombination solution at the point x_0 and rapidly breaks away and approaches the zero-current solution at the junction. The figure is schematic and has not been drawn to scale. In most cases of interest, the low fields in the recombination region will be much lower and the junction solution will hold over a smaller fraction of the intrinsic region.

It is convenient to take $x = 0$ not at the center x_i of the intrinsic region but at the minimum on the no-recombination solution. As the applied bias increases, x_i approaches zero.

Unequal Mobilities

In the general case of unequal mobilities, it is no longer so that I is pure drift except at the junctions. However we can define a linear com-

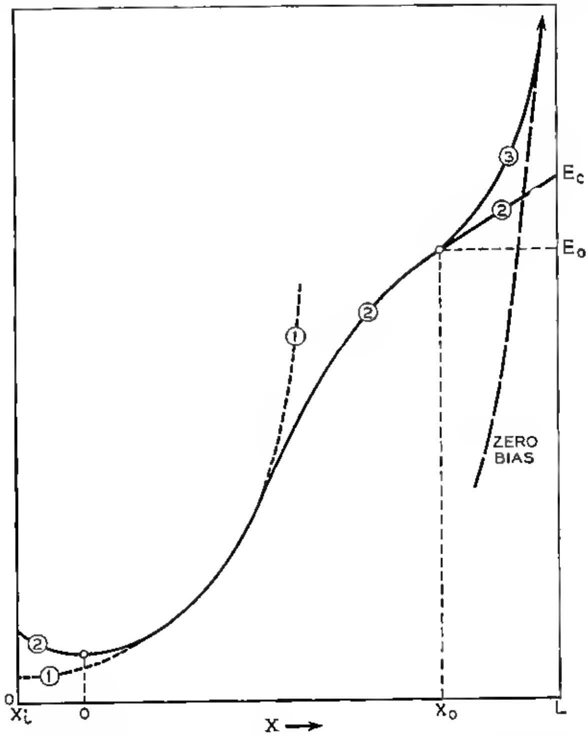

Fig. 1 — Schematic of the field distribution and the three overlapping solutions.

bination of J_p and J_n which has the same form as I in (2.6) and in which the diffusion term is negligible except near the junction. As we show in section VI, the effect of unequal mobilities is (1) to introduce some asymmetry into the curve in the region where the curvature is upward and (2) to displace the curve toward the NI junction (for the case where the electrons have the higher mobility). Thus the field is higher on the side where the carrier mobility is lower, as would be expected.

III. THE NO-RECOMBINATION CASE

This section deals with the case where recombination can be neglected in comparison with generation. This will be so where np is small compared to n_i^2.

The continuity equation for J now becomes

$$\frac{dJ}{dx} = 2g = \frac{n_i}{\tau} \tag{3.1}$$

Combining this witb (2.7) gives

$$\frac{d^2}{dx^2}\left(\frac{E^2}{E_1^2} - s\right) = \frac{1}{2L_i^2} \tag{3.2}$$

where $L_i^2 = D\tau$ is the diffusion length in intrinsic material.

Equation (3.2) can be immediately integrated. Tbere are two constants of integration, one of which can be made to vanish by cboosing $x = 0$ at the center of the intrinsic region, where tbe first derivatives of E and s vanish. (E is a minimum here and s a maximum). The solution obtained by two integrations is

$$\left(\frac{E}{E_1}\right)^2 - s = \left(\frac{x}{2L_i}\right)^2 - A \tag{3.3}$$

As we sball see later, tbe constant A is determined by the voltage drop across tbe unit.

The exact procedure now would be to substitute s from (3.3) into (2.6). The resulting second order differential equation could, in principle, then be solved for E versus x. The exact solution, bowever, would be quite difficult. We shall discuss it in Section V. Here we make the assumption that throughout tbe intrinsic region tbe cbarge flow is mainly by drift, so that we can neglect the diffusion term in (2.6) and take $I = \sigma s E$, as discussed in Section II. Later in this section we find an upper limit on the error due to this assumption and show how the cubic can be corrected to take account of the diffusion term.

Putting $s = I/\sigma E$ in (3.3) gives a cubic equation

$$\begin{aligned} \left(\frac{E}{E_1}\right)^2 - \frac{I}{\sigma E} &= \left(\frac{x}{2L_i}\right)^2 - A \\ \left(\frac{E}{E_1}\right)^2 - \frac{I}{\sigma E_1}\left(\frac{E_1}{E}\right) &= \left(\frac{x}{2L_i}\right)^2 - A \end{aligned} \tag{3.4}$$

for E/E_1 as a function of $x/2L_i$. This equation contains two parameters I and A. A determines the voltage and I is determined by the length $2L$ of the intrinsic region. The relation is as follows: Let the applied voltage drop across each junction be at least a few kT/q. Then tbe minority carrier currents from the extrinsic regions will have reached their saturation values. Call I_s the sum of the hole current from the N region and the electron current from the P region. I_s comes from pairs generated in the extrinsic regions near the junctions. I_s can be made arbitrarily small by making the N and P regions sufficiently highly doped (provided the diffusion length in the extrinsic material does not decrease with doping faster tban the majority carrier concentration in-

creases). The current generated in the intrinsic region is qg per unit volume. So the density of current from pairs generated in the intrinsic layer is $2Lqg = qn_iL/\tau$. Hence

$$I = I_s + \frac{qn_iL}{\tau}$$

In what follows we shall assume that I_s is negligibly small compared to I. Then

$$I = \left(\frac{qn_i}{\tau}\right)L = \left(\frac{qn_iD}{L_i}\right)\frac{L}{L_i} = \left(\frac{\sigma}{2L_i}\frac{kT}{q}\right)\frac{L}{L_i}$$

Thus I is L/L_i times a characteristic current equal to (1) the diffusion current produced by a gradient n_i/L_i or (2) the drift current produced by a field that gives the voltage drop kT/q in two diffusion lengths in normal intrinsic material. In germanium this characteristic current is about 5 milliamperes per cm^2.

That the current I is proportional to L and independent of voltage follows from the neglect of recombination. When recombination is small compared to generation, then the current has reached its maximum, or saturation, value. All the carriers generated in the intrinsic region are swept out before recombining. It will sometimes be convenient to take σE_1 as the unit of current. From the above and (2.8)

$$\frac{I}{\sigma E_1} = \frac{\sqrt{2}\,\mathcal{L}L}{(2L_i)^2} \tag{3.5}$$

In germanium σE_1 is about 7 amperes per cm^2. In general we will be dealing with currents that are small compared to this. For example, if L_i is 1 mm, we would have to sweep out an intrinsic region 3 meters long in order to get a current this large. If we take E_1 as the unit field, σE_1 as the unit current and $2L_i$ as the unit length then the cubic becomes $E^2 - I/E = x^2 - A$.

For a given structure and temperature the field versus x curves form a one parameter family. A determines both the field distribution and the voltage. The voltage increases as A decreases. Fig. 2 is a plot of E/E_1 versus $x/2L_i$ for $L/2L_i = 0.1$ and several different values of A. Fig. 3 is for $L = 2L_i$ and Fig. 4 for $L/2L_i = 3$.

There is an upper limit on A but not lower limit. The reason is as follows: As A increases, the minimum value of E (at $x = 0$) decreases and the maximum value of s increases. So if A is too large, the maximum s will be so large that we cannot neglect recombination, which becomes important when np approaches n_i^2, or s approaches 1. Fre-

quently recombination can be neglected over parts of the intrinsic region but not near the center, where the field is a minimum and the carrier concentration a maximum. Then (3.4) will represent the field distribution over that part of the region where recombination is unimportant. The correct solution will join onto the cubic as we move away from the center of the intrinsic region, which will no longer be at the $x = 0$ point on the cubic. In Section IV we solve the equations for the recombination region and show how the solution approaches the cubic. We will show that, as A increases, the distance from the center of the intrinsic region to the $x = 0$ point on the cubic also increases. The value $A = \frac{2}{3}$ corresponds to an infinitely long intrinsic region. For a larger A there exists no exact solution that could join onto the cubic as recombination becomes negligible. In Figs. 3 and 4 the $A = \frac{2}{3}$ curves join onto recombination solutions at values of E which are too low to show.

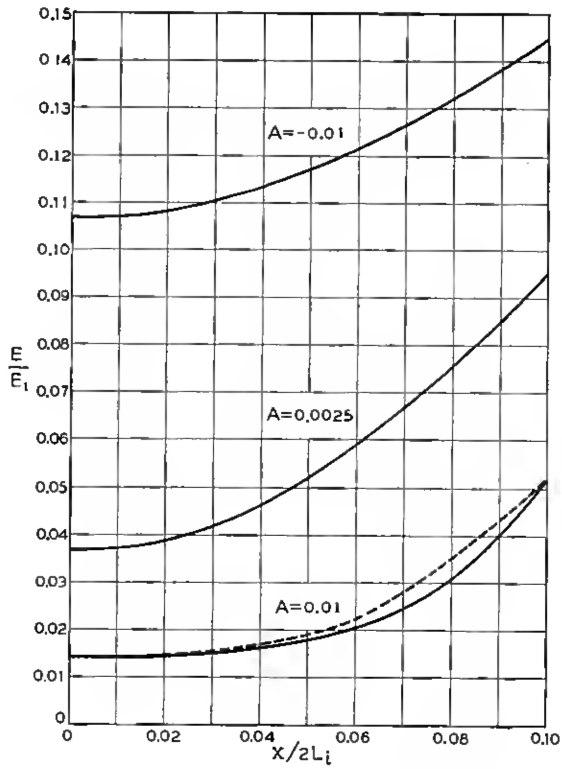

Fig. 2 — Field Distributions for $L = 0.2L_i$.

As A decreases and becomes negative the cubic approaches the form

$$E^2 = E_0^2 + E_1^2 \left(\frac{x}{2L_i}\right)^2 \tag{3.6}$$

where $E_0^2 = -AE_1^2$ is the minimum value of E^2. This form of the solution will be valid when the minimum E is large compared to (IE_1^2/σ). As E_0 increases, the voltage increases and the curve becomes flatter. This is because the increasing field sweeps the carriers out and reduces the space charge; so the drop in field decreases.

If (3.4) for E/E_1 versus $x/2L_i$ is extended to indefinitely large values of $x/2L_i$, it approaches the straight line of slope 1 going through the origin. Since E is always positive the curve is above this straight line at $x = 0$. If A is negative the curve is always above the straight line and always concave upward. If A is positive, the curve crosses the straight

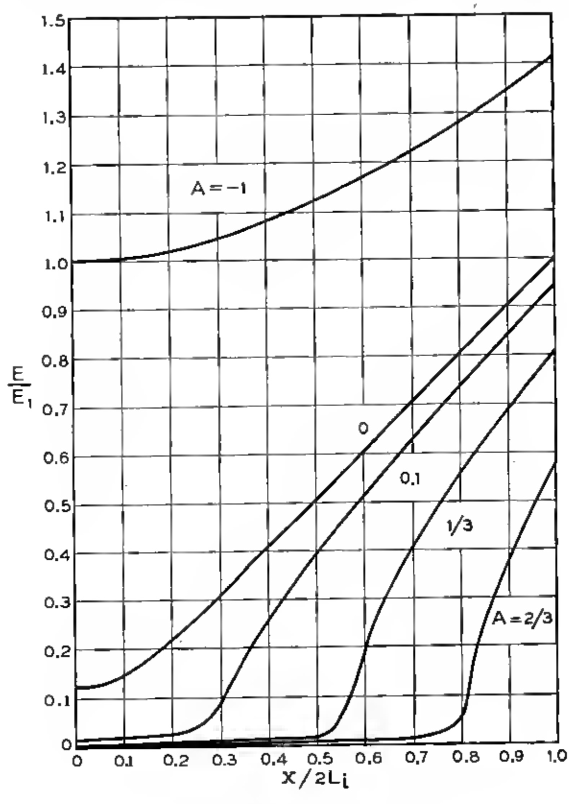

Fig. 3 — Field Distributions for $L = 2L_i$.

line at $E/E_1 = I/\sigma E_1 A$ and thereafter remains under it approaching it from below. For positive A the curvature, which is upward near the origin, changes to downward at about $x/2L_i = \sqrt{A}$.

The carrier concentrations n and p can be found from the E versus x curves with the aid of Poisson's equation $p - n = 1/a \, dE/dx$ and the definition $s = (n + p)/2n_i$ with $s = I/\sigma E$. These relations and (3.4) give

$$\frac{p - n}{p + n} = \frac{x}{L} \frac{1}{\left(1 + \dfrac{IE_1^2}{2\sigma E^3}\right)} \tag{3.7}$$

From (3.4) and (3.7) we may distinguish the following two regions on the cubic:

(1) When E^3/E_1^3 is smaller than $I/\sigma E_1$ (which as we have seen is usually smaller than unity), the E versus x curve is concave upward, the hole and electron concentrations are almost equal (charge neutrality) and the particle flow is by diffusion.

(2) When E^3/E_1^3 is greater than $I/\sigma E_1$, in general there is space charge and the particle flow, like the charge flow, is by drift. The curve is concave downward for positive A.

Figure 6, which we will discuss in Section IV, shows the field and carrier distributions for $L = 2L_i$ and $A = 0.665$ plotted on a logarithmic

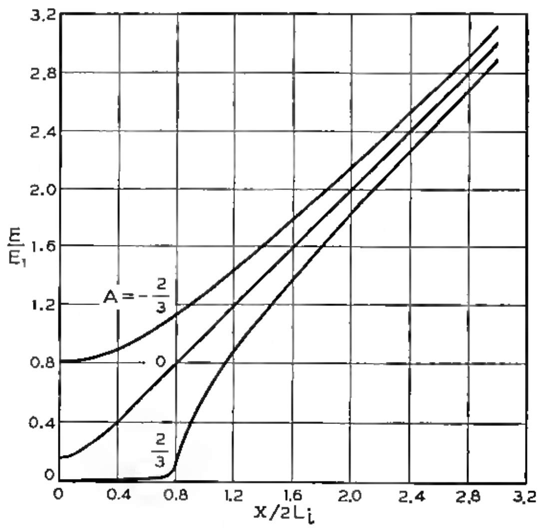

Fig. 4 — Field Distributions for $L = 6L_i$.

scale to show the behavior at low values of field and carrier density. In the region of no-recombination the field distribution is indistinguishable from that for $A = \frac{2}{3}$, which is plotted in Fig. 3 on a linear scale. In the region where recombination is important the solution is found from the assumption of charge neutrality as will be discussed in Section IV. The cubic and charge neutrality solutions are each shown dashed outside of their respective ranges of validity. For $A = 0.665$ the half length of the intrinsic region is $2.098 \times 2L_i$. Thus the length of the intrinsic region is more than twice the effective length $2L$ in which current is generated. The effective length will be discussed in more detail in Section IV and it will be shown that the effective length $2L$ of current generation is equal to the twice the distance from the IP junction to the minimum on the cubic. As explained earlier, it is convenient to take $x = 0$ at the minimum on the cubic.

Intrinsic-Extrinsic Junction Under Large Bias

Consider the limiting case of an intrinsic-extrinsic junction as the bias is increased and the space charge penetrates many diffusion lengths into the intrinsic material. Then the field distribution approaches the straight line $E/E_1 = x/2L_i$. This, by Poisson's equation, means that there is a constant charge density of N_i where

$$N_i = \frac{E_1}{2aL_i} = \frac{\sqrt{2}\mathfrak{L}}{L_i} n_i$$

Thus in the limit, the field in the intrinsic region approaches that in a completely swept extrinsic region having a fixed charge density of N_i. In germanium at room temperature N_i is about 4.10^{10} cm^{-3}. As the field approaches the limiting form, the voltage V approaches $E_1L^2/4L_i$. Thus the limiting form of the current voltage curve is

$$\frac{I}{\sigma E_1} = \frac{\mathfrak{L}}{L_i}\sqrt{\frac{V}{2E_1L}}$$

So in the limit the current varies as the square root of the voltage. Typical values for germanium at room temperature are $\sigma E_1 = 7$ amps cm^{-2}, $\mathfrak{L}/L_i = 10^{-3}$ and $2E_1L_i = 50$ volts.

Equivalent Generation Length for an Intrinsic-Extrinsic Junction

It should be noted that for an IP structure the current is the same as for an NIP structure with an infinite I region, or at least an I region that is long compared to the distance of penetration of the space charge.

Thus the equivalent length of current generation is $2L$ even though the current is actually being generated in an effective length L. The reason is that for an NIP structure the holes entering the right hand half of the I region were generated in the left hand side. For an IP structure the holes entering the space charge regions from the left were injected at the external left hand contact to the I region.

Applied Voltage

In all cases the voltage can be found from the area under the E versus x curve. In Figs. 2 to 4 the area under the curves gives the voltage accurately; recombination becomes important only where the field is so low as to have a negligible effect on the total voltage drop.

Correction of the Cubic

To conclude this section we consider the error introduced by using the assumption $I = \sigma sE$. For simplicity take E_1 as the unit field, $2L_i$ as the unit length and σE_1 as the unit current. Then the cubic becomes $E^2 - I/E = x^2 - A$. The corresponding exact solution is $E^2 - s = x^2 - A$ where the relation between s and E is given by equation (2.6) which in dimensionless form is

$$\mathfrak{L}^2 \frac{d^2E}{dx^2} = sE - I \tag{3.8}$$

where $\mathfrak{L}^2$ is of the order of 10^{-6}.

Let δE and δs represent the difference between the cubic and the correct solution at a given x. Assume that δE and its second derivative are small compared to E and its second derivative respectively. Then $\delta s = 2E\delta E$ and on the correct solution $sE - I = (2E^2 + I/E)\delta E$. So (3.8) becomes

$$\frac{\delta E}{E} = \left(\frac{\mathfrak{L}^2}{2E^3 + I}\right)\frac{d^2E}{dx^2} \tag{3.9}$$

To obtain a first approximation to $\delta E/E$ we use the cubic to evaluate d^2E/dx^2. It is convenient to express the results in terms of a dimensionless variable $z = E/I^{1/3}$, or if E and I are measured in conventional units $z = E(\sigma/E_1^2 I)^{1/3}$. Then (3.9) becomes

$$\frac{\delta E}{E} = \frac{1}{2}\left(\frac{L_i\mathfrak{L}}{L^2}\right)^{2/3}\left(\frac{z}{z^3 + \frac{1}{2}}\right)^2 + \left(\frac{x}{2L}\right)^2\frac{z^3(1 - z^3)}{(z^3 + \frac{1}{2})^4} \tag{3.10}$$

when the lengths are in conventional units.

The first term has a maximum value of 0.35 $(L_i\mathcal{L}/L^2)^{2/3}$ at $z = 0.6$ and tbe second term a maximum value of 0.18 at $z = 0.5$ and $x = L$.

The dashed curve in Fig. 2 for $A = .01$ is tbe corrected cubic. For the otber curves in Fig. 2, tbe correction is smaller. For tbe curves in Figs. 3 and 4 the correction is too small to sbow.

Limits on the Solution

We now show that δE as derived above is not only a first approximation but also upper limit on the correction necessary to take account of charge diffusion. That is, an exact solution to (3.8) lies between the cubic and the corrected cubic.

Consider the region where the second derivative of E is positive so that the perturbed curve lies above tbe cubic as in Fig. 2. Ou the cubic we bave $sE - I = 0$. As we move upward from the cubic and toward the dashed curve, $sE - I$ increases. Tbe value of $sE - I$ on the dashed curve just equals the value of $\mathcal{L}^2\, d^2E/dx^2$ on tbe cubic. However, the dasbed curve bas a smaller second derivative than the cubic. Thus in moving upward from the cubic toward the dashed curve $sE - I$ increases from zero and $\mathcal{L}^2\, d^2E/dx^2$, which is positive, decreases; on the dasbed curve $sE - I$ is actually greater. Therefore there is a curve lying just under the dashed curve where tbe two sides of (3.8) are equal. The same argument applied to the region where the second derivative is negative shows that the equation is satisfied by a curve lying just above the first perturbation of the cubic. Where the curvature cbanges sign, tbe cubic is correct.

It should be emphasized again that the neglect of the diffusion term in the current is justified only for the ideal case of equal hole and electron mobilities. For unequal mobilities both drift and diffusion will be important in current flow. However, as we will discuss in section 5, we can simplify the problem of unequal mobilities by defining a fictitious current that has the same form as I in (2.6) and (3.8).

IV. RECOMBINATION

As discussed in Section III, when the voltage for a given current is reduced, s increases and near $x = 0$ becomes comparable to unity. Then recombination becomes important and the cubic solution breaks down, or rather joins onto a solution that takes account of recombination. When recombination is important the center x_i of the intrinsic region is no longer at the $x = 0$ point on the cubic but to the left of it. That is, if we want the same current with continually decreasing voltage, we even-

tually get to the point where a longer intrinsic region is required. Finally for a given current we reach a minimum voltage which corresponds to an infinite length of intrinsic region. Another way of saying this is that, when recombination becomes important, the length L defined in terms of the current by $I = qg2L = qn_i/\tau L$ is no longer the half length of the intrinsic region.

Equivalent Generation Length

We shall continue to define L by $I = qn_i/\tau L$. Thus L is an equivalent, or effective, half length of current generation and not the half length of the intrinsic region. By definition L is the length such that the amount of generation alone in the length L is equal to the net amount of generation (generation minus recombination) in the total half length of the intrinsic region. Hence

$$gL = \int_{x_i}^{x_p} (g - r)\, dx \tag{4.1}$$

where x_i is at the center of the intrinsic region and x_p at the IP junction. We shall for the most part deal with reverse biases of at least a few kT/q, in which case recombination is negligible at the junctions. Then the exact solution becomes the no-recombination solution before reaching the junctions. We shall continue to take $x = 0$ at the point $dE/dx = ds/dx = 0$ on the no-recombination solution which the exact solution approaches as recombination becomes negligible.

Simplifying Assumptions

The general differential equation with recombination will be the same as for no-recombination except that $g - r$ replaces g. From (3.1) and (3.2)

$$\frac{d^2}{dx^2}\left(\frac{E^2}{E_1^2} - s\right) = \frac{1}{2L_i^2}\left(1 - \frac{r}{g}\right) \tag{4.2}$$

From (2.12) and (2.13) and Poisson's equation

$$\frac{r}{g} = \frac{np}{n_i^2} = \left(\frac{n + p}{2n_i}\right)^2 - \frac{(n - p)^2}{(2n_i)} = s^2 - 2\left(\frac{\mathcal{L}}{E_1}\frac{dE}{dx}\right)^2 \tag{4.3}$$

The following analysis will be based on the assumption of charge neutrality. That is we neglect terms in $p - n$ in comparison with those in $p + n$. In particular this means:

(1) The charge flows by drift so $I = \sigma s E$. This is the same assumption

made in tbe no-recombination case. It will be an even better approximation in tbe recombination region, wbere the second derivative of E is less.

(2) The particle flow is by diffusion. Tbat is, E^2/E_1^2 can be neglected in comparison with s.

(3) The ratio of recombination rate r to generation rate g is proportional to $g - r$; that is $g - r = g(1 - s^2)$.

All of these simplifying assumptions can be justified by substituting the resulting solution into the original expressions and showing that the neglected terms are small when recombination is important. If the solution is substituted into (4.3) and (2.6) tbe neglected terms will turn out to be negligible — and therefore assumptions (1) and (3), justified — wben s^2 is large compared to $\mathfrak{L}/L_i$. Assumption (2) follows from (1) and the fact that $I/\sigma E_1$ is small compared to unity.

Assumptions (2) and (3) may also be justified by tbe discussion following (3.7) in the following way: Where recombination is important s must be near uuity. So tbe cubic will begin to break down when $s = I/\sigma E$ becomes near to unity, or when E approacbes I/σ. However, if E is approximately I/σ tben $\sigma E^3/IE_1^2$ is approximately $(I/\sigma E_1)^2$, whicb, as we saw in the Section III, is small compared to unity in practical cases. Thus recombination becomes important and the solution joins onto tbe cubic in the range where E^3/E_1^3 is small compared to $I/\sigma E_1$. In this range the particle flow is by diffusion and $p - n$ is small compared to $p + n$. As we move toward the center of the intrinsic region s increases and E and dE/dx decrease. Therefore, since assumptions (2) and (3) are good wbere the solution joins onto the cubic, they are good throughout the region where recombination is important.

The Recombination Solution

The differential equation (4.2) now takes the form

$$\frac{d^2s}{dx^2} = -\frac{(1 - s^2)}{2L_i^2} \tag{4.4}$$

Tbe solution for s in the recombination range is seen to be the same for all values of the current. When s has been found E is found from $E = I/\sigma s$.

For small disturbances in normal carrier concentration, s is only slightly different from unity and (4.4) takes the familiar form

$$\frac{d^2}{dx^2}(1 - s) = \frac{1 - s}{L_i^2}$$

which says that the disturbance in carrier concentration varies exponentially as x/L_i.

Equation (4.4) can be integrated once to give

$$\left(\frac{ds}{dx}\right)^2 = \frac{1}{L_i^2}\left(s_0 - s - \frac{s_0^3 - s^3}{3}\right) \tag{4.5}$$

where s_0 is the value of s at the center of the intrinsic region where s is a maximum.

As recombination becomes unimportant, s^2 becomes small compared to unity and (4.5) approaches the form

$$\left(\frac{ds}{dx}\right)^2 = \frac{1}{L_i^2}\left[s_0\left(1 - \frac{s_0^2}{3}\right) - s\right] \tag{4.6}$$

and the solution joins onto the no-recombination solution.

Joining onto the Cubic.

We have seen that the solution joins onto the no recombination solution, in the region where particle flow is by diffusion so that the no recombination solution has the form $s = A - (x/2L_i)^2$. This is readily transformed to the form (4.6) with

$$A = s_0\left(1 - \frac{s_0^2}{3}\right) \tag{4.7}$$

Thus the one arbitrary parameter s_0 in the recombination solution determines the parameter A in the cubic that the recombination solution approaches. Since the maximum value of s_0 under reverse bias is unity, the maximum value of A is $\frac{2}{3}$. Negative values of A correspond to solutions where recombination is always negligible.

The s versus x Curve

To find s versus x we integrate (4.5). There is one constant of integration, which is fixed by the choice of $x = 0$. We have taken $x = 0$ at the point where $dE/dx = ds/dx = 0$ on the cubic. To make the recombination solution join the cubic we choose the constant of integration so that the recombination solution extrapolates to $s = 0$ at $(x/2L_i)^2 = A$. Then

$$\frac{x}{2L_i} = \sqrt{A} - \frac{\sqrt{3}}{2}\int_0^s \frac{ds}{\sqrt{3(s_0 - s) - (s_0^3 - s^3)}} \tag{4.8}$$

which can be expressed in terms of elliptic integrals.

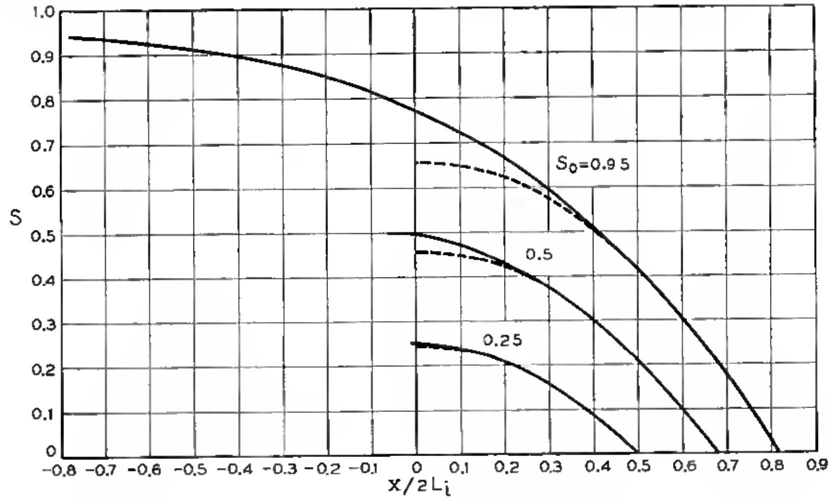

Fig. 5 — Variation of $s = p/n_i = n/n_i$ in the range where recombination is important.

Deep in an infinitely long intrinsic region the carrier densities approach their normal values $n = p = n_i$, or $s = 1$. Putting $s_0 = 1$ in (4.8), we find that as s approaches $s_0 = 1$, x becomes infinite. This will be the solution for a simple intrinsic-extrinsic junction. Fig. 5 is a plot of s versus x for various values of s_0. The dashed curves represent the corresponding no-recombination solution $s = A - (x/2L_i)^2$.

The IP Junction

It remains to find the position of the IP boundary. We now show that if recombination is unimportant at the junction, so that the solution joins onto a no-recombination solution, then the position of the junction is at $x = L$ where L is the effective length of current generation and $x = 0$ is the point where $dE/dx = ds/dx = 0$ on the no-recombination solution (which of course will not be valid at $x = 0$). The proof is as follows: From the definition (4.1) of L and (4.2)

$$L = \int_{x_i}^{x_p} (1 - r/g)\, dx = 2L_i^2 \int_{x_i}^{x_p} \frac{d^2}{dx^2}\left(\frac{E^2}{E_1^2} - s\right) dx$$
$$= 2L_i^2 \left[\frac{d}{dx}\left(\frac{E^2}{E_1^2} - s\right)\right]_{x=x_p} \tag{4.9}$$

If the boundary comes where recombination is negligible so that $(E/E_1)^2 - s = (x/2L_i)^2 - A$, then (4.9) gives $x_p = L$. Physically

this means that the amount of recombination in the interval from $x = 0$ to $x = L$ is just equal to the excess amount of generation in the interval from the center of the intrinsic region to $x = 0$.

If the applied reverse bias is less than a few kT/q then recombination is important even at the junction and there is no joining onto a no-recombination solution. In this case (4.9) says that for a given choice of current (and hence of L) the boundary comes where

$$\frac{ds}{dx} = -\frac{L}{2L_i^2} \tag{4.10}$$

Example. Fig. 6, which we discussed briefly in Section III, is a plot of the field and carrier distributions for $L = 2L_i$ and $s_0 = 0.95$, for which $A = 0.665$. The hole and electron densities were found from (3.7) and $p + n = 2n_i s$ where s is found from Fig. 5. When s approaches s_0 (4.8) for x versus s takes the simple form

$$\frac{x - x_i}{2L_i} = \frac{s_0 - s}{1 - s_0^2} \tag{4.11}$$

This will be accurate when $s_0 - s$ is small compared to $1/s_0 - s_0$. We have used (4.11) to evaluate the s versus x curve beyond the range of the $s_0 = 0.95$ curve in Fig. 5.

It is seen that the recombination solution in Fig. 6 joins the cubic in the range where n and p are still almost equal.

Variable Lifetime.

Finally consider the general case where the variation in τ with carrier density cannot be neglected. Then, with $n = p = n_i s$, (2.15) becomes $\tau = \tau_0 + (\tau_i - \tau_0)s$ and L_i^2 in (4.4) is replaced by $D\tau[1 + (\tau_i/\tau_0 - 1)s]$ where τ_i/τ_0 is given by (2.17). The more general form of (4.4) can be solved graphically after one integration. The solution will join onto a cubic if $(\tau_i/\tau_0 - 1)s$ becomes small compared to unity before space charge becomes important. This will be so if $(\tau_i/\tau_0 - 1)^{3/2} I/\sigma E_1$ is small compared to unity.

V. THE JUNCTION SOLUTION

In this section we consider the solution near the junctions, where the assumption $I = \sigma s E$ breaks down. We shall deal with reverse biases of at least a few kT/q so that recombination is negligible at the junctions. The junction solution will therefore join onto the no-recombination solution. We shall use the cubic solution in the no recombination region.

Again it is convenient to use dimensionless variables with E_1 as the

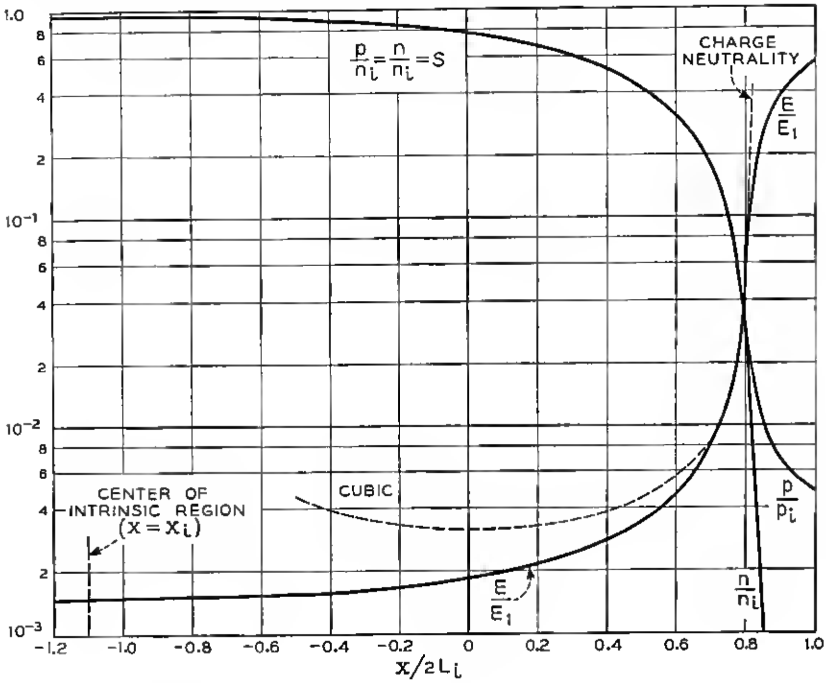

Fig. 6 — Field and carrier distributions for $L = 2L_i$ and $A = 0.665$ ($s_0 = 0.95$).

unit field, $2L_i$ as unit length and σE_1 as unit current. Then on the cubic $s = I/E$, and $E^2 - I/E = x^2 - A$. The current is related to L by $I = \sqrt{2}\mathfrak{L}L$ where the dimensionless $\mathfrak{L}$ is of the order of 10^{-3} for germanium at room temperature. Substituting the exact no-recombination solution $E^2 - s = x^2 - A$ into the solution (2.6), or (3.8), for the current gives the second order differential equation

$$\frac{d^2E}{dx^2} = \frac{1}{\mathfrak{L}^2}\left[E^3 - E(x^2 - A) - I\right] \tag{5.1}$$

for E as a function of x. The two boundary conditions are as follows: At $x = 0$, $dE/dx = 0$ by symmetry. At the IP junction the carrier concentration must rise and approach that in the normal P material. For a strongly extrinsic P region the normal hole concentration P is large compared to both n_i and the electron concentration. Thus s must increase and approach $P/2n_i \gg 1$ as we approach the P region. Clearly the cubic cannot satisfy this requirement. On the cubic the maximum value of s comes at $x = 0$ and is less than unity. As we approach the junc-

tion E increases so $s = I/E$ must decrease. Thus the correct solution must break away from the cubic near the junction.

Instability of the Solution

Equation (5.1) has two limiting forms and makes a rather abrupt transition between them. Over most of the intrinsic region, the quantity in brackets $[Es - I] = [E(E^2 - x^2 + A) - I]$ almost vanishes. It differs from zero just enough that when multiplied by the very large factor $\mathfrak{L}^2 \approx 10^6$ it gives the correct second derivative of E. In Section III we derived an upper limit on the small deviation δE from the cubic required to satisfy the differential equation. If E deviates from the cubic by more than this small amount, then the second derivative of E becomes too large. This increases the deviation from the cubic, which further increases the second derivative and so on. E and s rapidly approach infinity in a short distance. This, of course, is the required behavior at the junction. The rapid increase in s makes it possible for s to approach $P/2n_i$.

In Section III we showed that there is a solution to the differential equation that lies within a small interval δE from the cubic. Suppose we try to solve (5.1) graphically or on a machine starting at $x = 0$. There are two boundary conditions: By symmetry $dE/dx = 0$ at $x = 0$. We choose for $E(0)$ a value somewhere in the interval $\delta E(0)$. The resulting solution will not long remain in the interval $\delta E(x)$. In fact there is only one choice of $E(0)$ for which the solution remains close to the cubic from $x = 0$ to $x = \infty$. For any other $E(0)$ the solution would remain close to the cubic for a certain distance and then abruptly become unstable and both E and s approach infinity. $E(0)$ must be so chosen that the solution becomes unstable and E and s become large at the junction. However it is impractical to set $E(0)$ on a machine with sufficient accuracy to insure that the solution will remain stable for a reasonable distance. A more practical procedure is to find a solution which holds near the junction and joins the cubic to a solution in the adjacent extrinsic region.

Zero Bias

It may be helpful to approach the junction solution by reviewing the simple case of an IP junction under zero bias. Both charge and particle flow vanish. The vanishing of particle flow means that in the intrinsic region $E^2 - s$ is constant, (2.7). Since $E = 0$ and $s = 1$ in the normal intrinsic material, it follows that $E^2 - s = 1$. With $I = 0$ the equation

for current becomes

$$\frac{d^2E}{dx^2} = \frac{sE}{\mathfrak{L}^2} = \frac{E^3 + E}{\mathfrak{L}^2} \tag{5.2}$$

This can be integrated at once. The boundary conditions are $dE/dx = 0$ when $E = 0$ and $E = E_j$ at $x = L$; the field E_j at the IP junction will be determined by joining the solutions for the I and P regions. The solution can best be expressed by parametric equations giving x and the potential V as functions of E.

$$L - x = \mathfrak{L} \int_E^{E_j} \frac{dE}{E\sqrt{1 + E^2/2}} = \mathfrak{L}\left[\operatorname{csch}^{-1}\frac{E}{\sqrt{2}} - \operatorname{csch}^{-1}\frac{E_j}{\sqrt{2}}\right] \tag{5.3}$$

$$V_j - V = \mathfrak{L} \int_0^{E} \frac{dE}{\sqrt{1 + E^2/2}} = \frac{2kT}{q}\left[\sinh^{-1}\frac{E_j}{\sqrt{2}} - \sinh^{-1}\frac{E}{\sqrt{2}}\right] \tag{5.4}$$

where we have used the relation between dimensionless quantities $\mathfrak{L} = \sqrt{2}kT/q$, which follows from (2.8) with $E_1 = 1$. It will be more convenient to express voltages in terms of kT/q rather than in terms of the unit voltage $2E_1L_i$; then the ratio qV/kT is independent of the units. For convenience we take the voltage as increasing in going toward the IP junction with $V = 0$ in the normal P material. The voltage V_j at the junction is found by joining solutions.

On the P side, let the excess acceptor density be P. Adding the term $-aP$ to the right hand side of Poisson's (2.1), and proceeding as before we have, instead of (2.5)

$$\frac{d}{dx}\left(\frac{E^2}{E_1^2} - s - s_p\frac{qV}{kT}\right) = J = 0$$

where $s_p = P/2n_i$. We shall assume that the P region is strongly extrinsic so that $n \ll p$. Then $s = s_p$ in the normal p material, where $E = V = 0$. Hence

$$E^2 - s = s_p\left(\frac{qV}{kT} - 1\right) \tag{5.5}$$

In the intrinsic material the corresponding solution is $E^2 - s = -1$. Since both E and s are continuous at the junction, $qV_j/kT = 1 - 1/s_p$ where $1/s_p$ can be neglected. Thus $E_j^2 = s_j = s_p \exp[-(qV_j/kT)] = s_p/e$ where $e = 2.72$ is the base of the natural logarithms.

Knowing E_j we can find the field and potential distributions in the intrinsic material from (5.3) and (5.4).

Reverse Bias

Now in the intrinsic region, $E^2 - s = x^2 - A$. Let E_c be the value of E at the junction as given by the cubic, and let $s_c = I/E_c$ be the corresponding value of s. Then at the junction $x^2 - A = E_c^2 - s_c$. In the P material equation (5.5) will still be a good approximation near the junction, where the additional terms arising from tbe flow will be negligible. Joining the solutions for the I and P regions and neglecting s_c in comparison with s_p gives

$$\frac{qV_j}{kT} = 1 + \frac{E_c^2}{s_p}$$

Again using $s_j = s_p \exp[-(qV_j/kT)]$ we have

$$E_j^2 = E_c^2 + s_p \exp\{-(1 + E_c^2/s_p)\} \tag{5.6}$$

In most practical cases E_c^2 will be small compared to $s_p = P/2n_i$ so E_j will be the same as for zero bias.

Junction Solution

We now consider an approximate solution that joins smoothly onto the cubic and has the required behavior at the junction. Let $x = x_0$ be the point where this solution is to join the cubic. Then in (5.1) x^2 must lie between x_0^2 and L^2. We can obtain two limiting forms of the solution by giving x the two constant values, x_0 and L respectively. It will be best to take $x = x_0$ since in practical cases the x^2 term is not important except near the point where the junction solution joins the cubic. In all cases the uncertainty due to taking x^2 = constant can be estimated by comparing the solutions for $x = x_0$ and $x = L$.

With x^2 constant, (5.1) can easily be integrated. The two boundary conditions are (a) $E = E_j$ at $x = L$, where E_j is given by (5.6), and (b) to insure a smooth joining, the slope at $x = x_0$ must be the same as that of the cubic, namely

$$\left(\frac{dE}{dx}\right)_0 = \frac{2x_0}{2E_0 + I/E_0^2} \tag{5.7}$$

The first integration of (5.1) with $x = x_0$ gives

$$\left(\frac{dE}{dx}\right)^2 = \left(\frac{dE}{dx}\right)_0^2 + \frac{2}{\mathfrak{L}^2}\left[\frac{E^4}{4} - \frac{E^2}{2}(E_0^2 - I/E_0) - IE\right]_{E_0}^{E} \tag{5.8}$$

where (dE/dx) is given by (5.7) and $E_0^2 - I/E_0 = x_0^2 - A$. The E versus

x curve can now be found from (5.8) and

$$\begin{aligned} x - x_0 &= \int_{E_0}^{E} \left(\frac{dE}{dx}\right)^{-1} dE \\ L - x &= \int_{E}^{E_j} \left(\frac{dE}{dx}\right)^{-1} dE \end{aligned} \tag{5.9}$$

In general we will be intrested in cases where the junction solution holds over a length $L - x_0$ that is small compared to L, so we can take $x_0 = L$ in (5.7). It will also be valid to let E_0 in (5.7) and (5.8) be the value E_c on the cubic at $x = L$. Putting $E_c = E_0$ in equation (5.6) then gives E_j in terms of E_0 and $s_p = P/2n_i$, where P is the majority carrier concentration in the extrinsic region. In what follows we shall use these approximations. It will be convenient to express $x_0 = L$ in (5.7) in terms of I using $I = \sqrt{2}L\mathfrak{L}$. We continue to use dimensionless quantities with E_1, $2L_i$ and σE_1 as the units of field, length and current respectively, and $2L_iE_1$ as the unit of voltage. In general however we can express voltages in terms of kT/q.

When E_0^3 is either large or small compared to I, the junction solution takes a simple form and the field and potential distributions can he found analytically. We next consider two approximations that bold in those two cases respectively. Relatively good agreement between the two solutions at $E_0^3 = I$ indicates that each solution may be used up to this point.

Case of E_0^3 Large Compared to I

From (5.7) to (5.9)

$$x - x_0 = \sqrt{2}\mathfrak{L} \int_{E_0}^{E} \left[\left(\frac{I}{E_0}\right)^2 + (E^2 - E_0^3)^2\right]^{-1/2} dE \tag{5.10}$$

This can be solved in the following two overlapping ranges where the integrand has a simple form:

Range 1. Here $E - E_0$ is small compared to $2E_0$, so (5.10) becomes

$$x - x_0 = \frac{\mathfrak{L}}{\sqrt{2E_0}} \sinh^{-1}\left[(E - E_0)\frac{2E_0^3}{I}\right] \tag{5.11}$$

Since E and E_0 are almost equal, we have for the voltage drop in this range

$$V - V_0 = E_0(x - x_0) \tag{5.12}$$

Range 2. Here $E^2 - E_0^2$ is large compared to $2(\mathfrak{L}L/E_0)^2$, so (5.10) gives

$$L - x = \sqrt{2}\mathcal{L} \int_E^{E_j} \frac{dE}{E^2 - E_0^2} = \frac{\sqrt{2}\mathcal{L}}{E_0} \left(\operatorname{ctnh}^{-1} \frac{E}{E_0} - \operatorname{ctnh}^{-1} \frac{E_j}{E_0} \right) \tag{5.13}$$

From $E_0^3 \gg I$ it follows that Ranges 1 and 2 overlap. By joining the two solutions in the overlap region, the solution in Range 2 can be written as

$$x - x_0 = \frac{\mathcal{L}}{\sqrt{2}\, E_0} \ln \left[\frac{8E_0^3}{I} \frac{E - E_0}{E + E_0} \right] \tag{5.14}$$

Putting $E = E_j$ in (5.14) gives the distance over which the junction solution holds. In general we will be interested in cases where E_j is large compared to E_0 so (5.14) becomes

$$\frac{L - x_0}{l} = \frac{3}{2} \frac{\ln 2z_0}{z_0} \tag{5.15}$$

where $l = \sqrt{2}\mathcal{L}/I^{1/3}$ and as before $z_0 = E_0/I^{1/3}$. In conventional units

$$l = 2L \left(\frac{\mathcal{L}L_i}{L^2} \right)^{2/3} \tag{5.16}$$

Fig. 7 is a plot of $(L - x_0)/l$ versus z_0. In germanium at room temperature $\mathcal{L}L_i$ will be around 10^{-5} cm. Thus the junction solution will hold over a region that is small compared to L if L is large compared to 3×10^{-3} cm.

Again it is convenient to use the relation $\mathcal{L} = \sqrt{2}kT/q$ to express the voltage in terms of kT/q.

$$V_j - V = \int_E^{E_j} E \left(\frac{dE}{dx} \right)^{-1} dE = \frac{kT}{q} \ln \left(\frac{E_j^2 - E_0^2}{E^2 - E_0^2} \right) \tag{5.17}$$

By joining the two solutions in the overlap region, the voltage in Range 2 can be expressed as

$$V - V_0 = \frac{kT}{q} \ln \left(\frac{2E_0}{I} \right) (E^2 - E_0^2) \tag{5.18}$$

Setting $V = V_j$ and $E = E_j$ in (5.18) gives the total voltage drop in the region where the junction solution holds. Let ΔV be the difference between $V_j - V_0$ and the built in voltage drop at the junction. Then substituting (5.6) with $E_c = E_0$ into (5.18) and subtracting the built in drop we have for ΔV,

$$\Delta V = \frac{kT}{q} \left[\ln \frac{E_0}{I} - \frac{E_0^2}{s_p} \right] \tag{5.19}$$

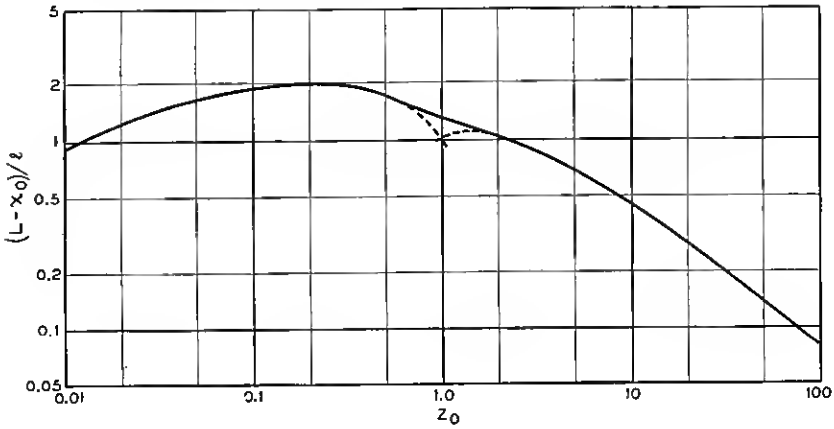

Fig. 7 — Variation of $(L - x_0)/\ell$ with z_0 .

I/E_0 is equal to the value of s on the cubic at $x = L$. For positive values of A the maximum value of E_0/I is $L/I = 1/\sqrt{2}\mathfrak{L}$ as can be seen from the cubic. In germanium at room temperature $\mathfrak{L}$ is about 10^{-3} (for $2L_i$ = unit length) so the reverse bias produces an additional voltage drop in the junction region equal to about $7kT/q$. For negative values of A the additional voltage drop near the junction would be higher.

Comparing (5.3) and (5.13) we see that the junction solution reduces to the zero bias solution when E^2 is large compared to $E_0^2 + 2$. In this case both solutions have the simple form

$$L - x = \sqrt{2}\mathfrak{L}\left(\frac{1}{E} - \frac{1}{E_j}\right) \tag{5.20}$$

and

$$V_j - V = \frac{2kT}{q}\,\ell n\,\frac{E_j}{E} \tag{5.21}$$

Case of E_0^3 Small Compared to I

Now from (5.7) and (5.8) with $x_0 = L = I\sqrt{2}\mathfrak{L}$ we have

$$\begin{aligned} \left(\frac{dE}{dx}\right)_0^2 &= \left(\frac{2LE_0^2}{I}\right)^2 \\ \left(\frac{dE}{dx}\right)^2 &= \frac{1}{\mathfrak{L}^2}\left[2E_0^4 + (E - E_0)^2\left(\frac{E^2}{2} + \frac{I}{E_0}\right)\right] \end{aligned} \tag{5.22}$$

Again there are two overlapping ranges where the solution has a simple form:

Range 1. Here E^2 is small compared to $2I/E_0$. This will be so even when E becomes large compared to E_0. Setting $c_1^2 = 2E_0^5/I$ and $y = E - E_0$ in equation (5.22) and integrating gives

$$
\begin{aligned}
x - x_0 &= \mathfrak{L}\sqrt{\frac{E_0}{I}}\int_0^{E-E_0}\frac{dy}{\sqrt{c_1^2 + y^2}} \\
&= \mathfrak{L}\sqrt{\frac{E_0}{I}}\sinh^{-1}\left(\frac{E - E_0}{c_1}\right)
\end{aligned}
\tag{5.23}
$$

and

$$
\begin{aligned}
&V - V_0 \\
&= \frac{kT}{q}\sqrt{\frac{2E_0}{I}}\left(\sqrt{c_1^2 + (E - E_0)^2} - c_1\right) + 2E_0(x - x_0)
\end{aligned}
\tag{5.24}
$$

Range 2. Here E is large compared to E_0. It follows from $E_0^3 \ll I$ that E is also large compared to c_1. Setting $c_2^2 = 2I/E_0$ we have

$$
\begin{aligned}
L - x &= \sqrt{2}\mathfrak{L}\int_E^{E_j}\frac{dE}{E\sqrt{E^2 + c_2^2}} \\
&= \mathfrak{L}\sqrt{\frac{E_0}{I}}\left(\operatorname{csch}^{-1}\frac{E}{c_2} - \operatorname{csch}^{-1}\frac{E_j}{c_2}\right)
\end{aligned}
\tag{5.25}
$$

Joining (5.21) and (5.23) where they overlap we have in range (2)

$$
x - x_0 = \mathfrak{L}\sqrt{\frac{E_0}{I}}\,\ell n\left(\frac{2I}{E_0^3}\right)\left[\frac{E}{c_2 + \sqrt{c_2^2 + E^2}}\right] \tag{5.26}
$$

Putting $x = L$ and $E = E_j$ in (5.26) gives the length $L - x_0$ in which the junction solution holds. If E_j is large compared to c_2, then

$$
\frac{L - x_0}{l} = \sqrt{\frac{z_0}{2}}\,\ell n\,\frac{4}{z_0^3} \tag{5.27}
$$

where as before $z_0 = E_0/I^{1/3}$ and l is given by (5.16). Fig. 7 is a plot of $(L - x_0)/l$ versus z_0. The two approximations (5.15) and (5.27) for $z_0^3 \ll 1$ and $z_0^3 \gg I$ respectively are shown dashed. Both become inaccurate as they are extended toward $z_0 = 1$. The point at $z_0 = 1$ was obtained graphically. Each approximation is in error by about 28 per cent here. The error will decrease as each approximation is extended away from $z_0 = 1$ toward its range of validity.

The voltage in Range 2 is given by

$$V_j - V = \frac{2kT}{q}\left[\sinh^{-1}\frac{E_j}{c_2} - \sinh^{-1}\frac{E}{c_2}\right] \tag{5.28}$$

or again joining (5.28) to the solutions in Range 1 we have in Range 2

$$V - V_0 = \frac{2kT}{q}\sinh^{-1}\frac{E}{c_2} + 2E_0(x - x_0) \tag{5.29}$$

The total voltage drop in the junction can be found by setting $V = V_j$ and $E = E_j$ in (5.29). The term $2E_0(L - x_0)$ will be negligible. When E^2 is large compared to $c_2^2 + 2$ the junction solution reduces to the zero current solution as can be seen by comparing (5.3) and (5.25). Then the solution has the simple form (5.20) and (5.21). E_j will always be large compared to c_2. (E_j^2 is approximately s_p/e and $c_2^2 = 2s_0$ where s_0 is the value of s where the junction solution joins the cubic.) Thus the difference ΔV between $V_j - V_0$ and the built in voltage is

$$\Delta V = \frac{kT}{q}\,\ell n\,\frac{E_0}{I} \tag{5.30}$$

Example. Fig. 8 shows the field distribution near the IP junction for the case $L = 2L_i$ and $A = \frac{2}{3}$, for which the intrinsic region is infinitely long. The field distribution near the junction, however, will be indistinguishable from that for $A = 0.665$, or $s_0 = 0.95$, for which the intrinsic region is about twice the effective length of current generation. We have taken $E_j = 30$, which corresponds to an excess acceptor density $P = 4.7 \times 10^3\, n_i$ in the P region. Over the range where the junction solution holds the cubic gives an almost constant field $E = E_0 = E_c$. The junction solution goes from the cubic to the zero bias solution in a distance of the order of the Debye length. The sum of the built in voltage and the voltage derived from the cubic differ from the correct voltage by the order of $\mathfrak{L}E_1$ or about kT/q. The total voltage is about $0.3\, E_1 L_i$, which would be about 11 volts in germanium at room temperature.

VI. GENERAL CASE, UNEQUAL MOBILITIES

This Section deals with the general case where the ratio of the hole and electron mobilities is arbitrary. The procedure is similar to that used in the preceding Sections. Many of the results for $b = 1$ are useful in the present, more general, case. We shall deal first with the no-recombination case and again find that E is given by a cubic. However, the field distribution is no longer symmetrical and the coefficient of the I/E term in the cubic is a linear function of x instead of a constant. The differential equation for s in the recombination region remains un-

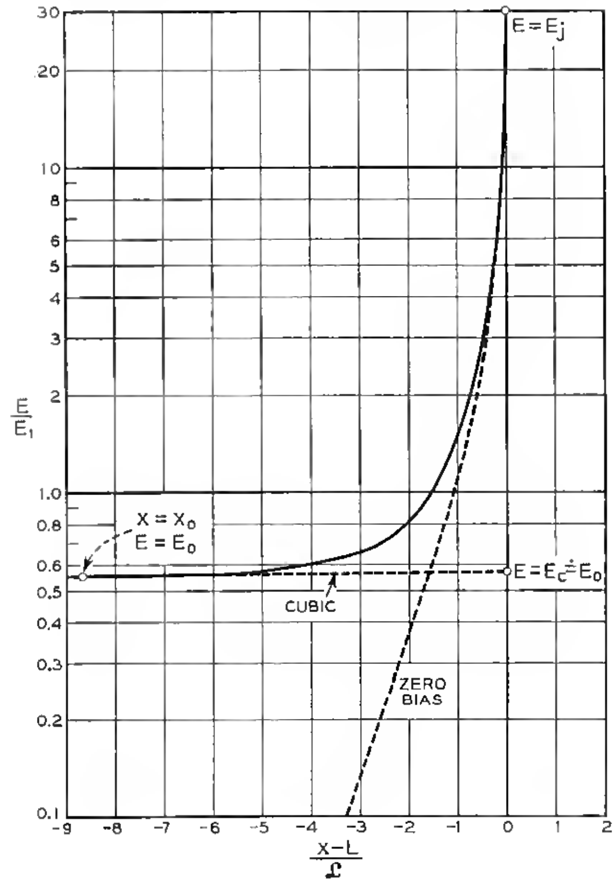

Fig. 8 — Field Distribution near the IP Junction for $L = 2L_i$ and $A = \frac{2}{3}$.

changed. It is no longer so that charge diffusion can be neglected except near the junctions. However, there is a linear combination of J_p and J_n in which the diffusion term is negligible except near the junctions.

Basic Relations

The equations are the two continuity (2.9) and Poisson's (2.1). The formulas for $g - r$ remain unchanged, since they involve only the statistics of recombination and are independent of mobility. The hole and electron currents are given by (2.2) with b arbitrary. Equation (2.2) for J_p in terms of E, p and n remains unchanged. Now J_n/b has the same

form as J_n had for the $b = 1$ case. It is therefore desirable to deal with the fictitious carrier flow $J_p + J_n/b$ and the fictitious current $q(J_p - J_n/b)$ since these have the same form in terms of E and $s = (n + p)/2n_i$ as J and I had for $b = 1$. Thus

$$J_p + \frac{J_n}{b} = 2n_iD \frac{d}{dx}\left(\frac{E^2}{E_1^2} - s\right) \tag{6.1}$$

$$q\left(J_p - \frac{J_n}{b}\right) = \frac{2}{1 + b}\,\sigma\left[Es - \mathfrak{L}^2 \frac{d^2E}{dx^2}\right] \tag{6.2}$$

where E_1 and $\mathfrak{L}$ have the same meaning as before and the conductivity of intrinsic material is now $\sigma = qn_i\mu(1 + b)$. As before D and μ are respectively the diffusion constant and mobility for holes. Equations (6.1) and (6.2) reduce respectively to (2.7) for J and (2.6) for $I = q(J_p - J_n)$ where $b = 1$.

When the flow is by pure diffusion, the holes and electrons diffuse "in parallel" so the effective diffusion constant is the reciprocal of the average of the reciprocal hole and electron diffusion constants. Hence the effective diffusion length is given by

$$L_i^2 = D\tau \frac{2b}{1 + b} \tag{6.3}$$

We continue to let $2L = I/qg$ be the effective length of current generation; again it is the actual length for the no recombination case. Let x_n and x_p be the coordinates of the NI and IP junctions respectively. Since the problem is not symmetrical we will not take $x = 0$ in the center of the intrinsic region even for the no-recombination case.

No-Recombination Case

Setting $r = 0$ we can immediately integrate the continuity equations

$$\frac{dJ_p}{dx} = \frac{dJ_n}{dx} = g$$

subject to the boundary conditions:

$$\begin{aligned} &\text{at the } NI \text{ junction, } x = x_n, \quad && J_p = 0, \quad && J_n = -I/q \\ &\text{at the } IP \text{ junction, } x = x_p, \quad && J_p = I/q, \quad && J_n = 0 \end{aligned} \tag{6.4}$$

The result is $J_p = g(x - x_n)$ and $J_n = g(x - x_p)$. This agrees with $I = q(J_p - J_n) = 2qgL$ since $2L = x_p - x_n$ is the length of the intrinsic region, which, for no-recombination, is also the effective length of cur-

rent generation. It will be convenient to choose $x = 0$ so that $x_n = -x_p/b$. Then the origin is nearer to the NI junction for $b > 1$. Now from this and the boundary conditions (6.4) and $I = 2qgL$ we have the positions of the junctions:

$$\frac{x_p}{L} = \frac{2b}{1+b}, \qquad \frac{x_n}{L} = \frac{-2}{1+b} \tag{6.5}$$

As before, the junctions are at $x = \pm L$ for $b = 1$.

We can now find the fictitious carrier flow $J_p + J_n/b$ and the fictitious current $q(J_p - J_n/b)$ as functions of x.

$$J_p + \frac{J_n}{b} = \left(\frac{1+b}{b}\right) gx \tag{6.6}$$

$$q\left(J_p - \frac{J_n}{b}\right) = \frac{2I}{1+b}\left(1 + \frac{\beta x}{L}\right) \tag{6.7}$$

where the dimensionless parameter $\beta = (b^2 - 1)/4b$. Thus the fictitious current $q(J_p - J_n/b)$ is equal to the actual current times a linear function of x. This function is always positive and varies from a minimum of $1/b$ to a maximum of 1.

Combining (6.6) with (6.1) and integrating gives the equation

$$\frac{E^2}{E_1^2} - s = \left(\frac{x}{2L_i}\right)^2 - A \tag{6.8}$$

that we had before. Now, however, E is not a minimum at the same point where s is a maximum. As before, when recombination is negligible throughout all of the intrinsic region, A determines the voltage; and, when recombination is important over part of the region, A determines both the voltage and the length of the intrinsic region $x_p - x_n > 2L = I/gq$.

Combining (6.7) with (6.2) gives

$$I\left(1 + \frac{\beta x}{L}\right) = \sigma\left[Es - \mathfrak{L}^2 \frac{d^2E}{dx^2}\right] \tag{6.9}$$

which is similar to the previous (3.6) except that I is multiplied by the factor $1 + \beta x/L$, which varies from $1 + 1/b$ to $1 + b$. The same arguments used in Section V apply here and show that the second term in brackets (the diffusion term) can be neglected except near the junctions. In other words, although I is always part drift and part diffusion, $I(1 + \beta x/L)$ is approximately pure drift except at the junctions.

Eliminating s between (6.9) and (6.8) and neglecting the diffusion

term in (6.9) gives the cubic equation

$$\frac{E^2}{E_1^2} - \frac{I}{\sigma E}\left(1 + \frac{\beta x}{L}\right) = \left(\frac{x}{2L_i}\right)^2 - A \tag{6.10}$$

for the field distribution.

In germanium, where $b = 2.1$, $\beta = 0.406$, $x_p = 1.35L$ and $x_n = -0.65L$. The coefficient of $I/\sigma E_1$ therefore varies from 1.47 to 3.10, or by a factor of a little more than 2. This will introduce some asymmetry into the E versus x curve in the low field region where the fictitious carrier flow $J_p + J_n/b$ is by diffusion. It is evident that, as the voltage increases, the field versus x curve becomes increasingly symmetrical about the $x = 0$ point; so the effect of having $b \neq 1$ is simply to shift the field distribution along the x axis.

Recombination

The arguments of section 4 again apply. Where recombination is important, $n - p$ is small compared to $n + p$, so $g - r = g(1 - s^2)$. The diffusion term dominates in the fictitious particle flow $J_p + J_n/b$; that is, E^2/E_1^2 is small compared to s, so (6.1) becomes

$$J_p + \frac{J_n}{b} = -2n_i D \frac{ds}{dx}$$

The continuity equations give

$$\frac{d}{dx}\left(J_p + \frac{J_n}{b}\right) = \left(1 + \frac{1}{b}\right)(g - r) = \frac{n_i(1 + b)}{2\tau b}(1 - s^2)$$

So again we have

$$\frac{d^2 s}{dx^2} = -\frac{(1 - s^2)}{2L_i^2} \tag{6.11}$$

The solution joins the no recombination solution where $s = A - (x/2L_i)^2$. Therefore A is again related to s_0, the maximum s, by $A = s_0(1 - s_0^2/3)$ and the s versus x curve is given by (4.8) and is symmetrical about the point where s is a maximum. When the recombination solution joins onto no-recombination solutions, there will be a different no-recombination solution on each side of the recombination region. The junctions will be at the points x_p and x_n on the respective no-recombination solutions. The length of the intrinsic region will not be $x_p - x_n = 2L$ since the $x = 0$ points are different on the two no-recombination solutions and are separated by a region of maximum recombination.

To find E when s is known we express the current $I = q(J_p - J_n)$ in terms of s and E. Since $n - p$ is small compared to $n + p$, we set $n = p = sn_i$ in (2.2) and obtain

$$I = \sigma\left[sE - \frac{1-b}{1+b}\frac{kT}{q}\frac{ds}{dx}\right] \tag{6.12}$$

Thus the current contains both a drift and a diffusion term. This is to be expected for unequal mobilities. When holes and electrons have the same concentration gradient, the electrons, which have the higher diffusion constant, diffuse faster than the holes; hence the diffusion gives a net current. It is seen that in the recombination region the total carrier concentration has a symmetrical distribution about the point where it is a maximum but the field remains unsymmetrical.

Junction Solution

When $(E_0/E_1)^3$ is large compared to $I/\sigma E_1$ the junction solution is independent of b; so the solution obtained in Section V is valid. In all cases the junction solution can be found using the method of Section V. The effect of b will be small over most of the range where the junction solution holds because the concentration of one type of carrier will be negligible. To be exact, I in (5.8) should be multiplied by the factor $(1 + \beta x_0/L)$, which can be taken to be $(1 + b)/2b$ at the NI junction and $(1 + b)/2$ at the IP junction. Instead of equation (5.7) we have

$$\left[2E_0 + \frac{I}{E_0^2}\left(1 + \frac{\beta x_0}{L}\right)\right]\left(\frac{dE}{dx}\right)_0 = 2x + \frac{I}{E_0}\frac{\beta}{L} \tag{6.13}$$

as can be seen by differentiating (6.10) with $E_1 = 2L_i = \sigma = 1$.

VII. EFFECT OF FIXED CHARGE

This section will deal briefly with the case where there is some fixed charge but where the carrier charge cannot be neglected. For no recombination, the field distribution is given by a first order differential equation. Solutions in closed form are obtained for the case of pure drift flow. For recombination and charge neutrality the solution in Section IV is valid provided the fixed charge is small compared to n_i. We have seen that at large fields the E versus x curve becomes linear, corresponding to a fixed charge density of N_i where $N_i = \sqrt{2}n_i\mathfrak{L}/L_i$. Thus the fixed charge may have a dominant effect on the space charge while having a negligible effect on the solution in the range where recombination is important.

Let the density of fixed charge be $N = N_d - N_a$ = excess density of donors over acceptors. N may be either positive or negative. In what follows we shall assume that N is positive. So the structure is $N\nu P$ where ν means weakly doped n-type. Equations (2.2) for the hole and electron currents remain unchanged. Poisson's equation becomes

$$\frac{dE}{dx} = a(p - n + N) \tag{7.1}$$

We shall deal with the general case of arbitrary mobilities. As in Section VI it is convenient to deal with a fictitious current $q(J_p - J_n/b)$ and a fictitious particle flow $J_p + J_n/b$. The extra term in (7.1) drops out by differentiation when (7.1) is substituted into the equation for $J_p - J_n/b$ so (6.2) remains unchanged. However, instead of (6.1) we have

$$J_p + \frac{J_n}{b} = 2n_i D \frac{d}{dx}\left(\frac{E^2}{E_1^2} - s\right) - \mu N E \tag{7.2}$$

So the fictitious particle flow is no longer the gradient of a potential involving only E and s.

No Recombination

As in Section VI the continuity equations can be immediately integrated to give (6.6) and (6.7). Again I is given by (6.9) where the diffusion term on the right can be neglected except at the junctions; so again we have $\sigma s E = I(1 + \beta x/L)$. Substituting this into (7.2) and combining (7.2) and (6.6) gives a first order differential equation for E versus x. It is convenient to again use dimensionless quantities with E_1, $2L_i$ and σE_1 as the units of field, length and current respectively. Then the differential equation becomes

$$\frac{d}{dx}\left[E^2 - \frac{I}{E}\left(1 + \frac{\beta x}{L}\right)\right] = 2(x + \alpha E) \tag{7.3}$$

where

$$\alpha = \frac{N}{N_i}$$

and as before $N_i = \sqrt{2}n_i\mathfrak{L}/L_i$, which is around 4×10^{10} in germanium at room temperature. The solution of (7.3) contains one arbitrary constant (which corresponds to A in the $N = 0$ case). The lower limit on the constant is determined by the necessity of joining onto a recombination solution where s approached unity. The positions of x_n and x_p of the $N\nu$ and νP junctions respectively are given by (6.5).

In the region of low fields where E^3 is comparable to or less than I, (7.3) would have to be solved graphically or on a machine. At higher fields the equation is easily integrated as discussed below.

Case of Pure Drift

When the flow is entirely by drift, $E^3 \gg I$ and (7.3) becomes

$$\frac{dE}{dx} = \frac{x}{E} + \alpha \tag{7.4}$$

which is made integrable by the substitution $E = yx$. A family of solutions for positive E throughout the ν region is

$$(E - a_1x)^{a_1}(E + a_2x)^{a_2} = E_0^{a_1+a_2} \tag{7.5}$$

where $2a_1 = \sqrt{4 + \alpha^2} + \alpha$ and $2a_2 = \sqrt{4 + \alpha^2} - \alpha$ and E_0 is the value of E at $x = 0$. For an intrinsic region $N = \alpha = 0$ and (7.5) reduces to $E^2 = E_0^2 + x^2$, which is the same as (3.9) for negative A. Fig. 9 shows several curves for various values of E_0. These remain above, and at large distances approach, the asymptotic solutions $E = a_1x$ on the right of the origin and $E = -a_2x$ on the left. These curves differ from the corresponding curves for an intrinsic region in that the straight line asymptotes now have slopes of a_1 and $-a_2$ instead of ± 1. Toward the P side the slope is greater because the positive change qN of the excess donors is added to the charge of holes. Toward the N side of the ν region the slope is reduced because N compensates to some extent for the electron charge. As α increases and the ν region becomes more n type, the solution approaches that for a simple NP junction, where $E = \alpha x$ on the N side.

Another set of solutions of (7.4) are given by

$$(a_1x - E)^{a_1}(a_2x + E)^{a_2} = a_1^{a_1}a_2^{a_2}x_c^2 \tag{7.6}$$

Several of these are shown in Fig. 9. They remain below the linear asymptotes and go through zero field at $x = \pm x_c$. Actually these will join onto solutions of the more general equation (7.3) when E becomes small and the diffusion term becomes important.

Recombination. When the fixed charge density is small compared to the intrinsic hole and electron density the treatment of recombination in Section IV remains valid. The recombination solution joins onto a solution of (7.3) at small fields. When N is comparable to n_i the recombination solution is difficult even with the assumption of charge neutrality.

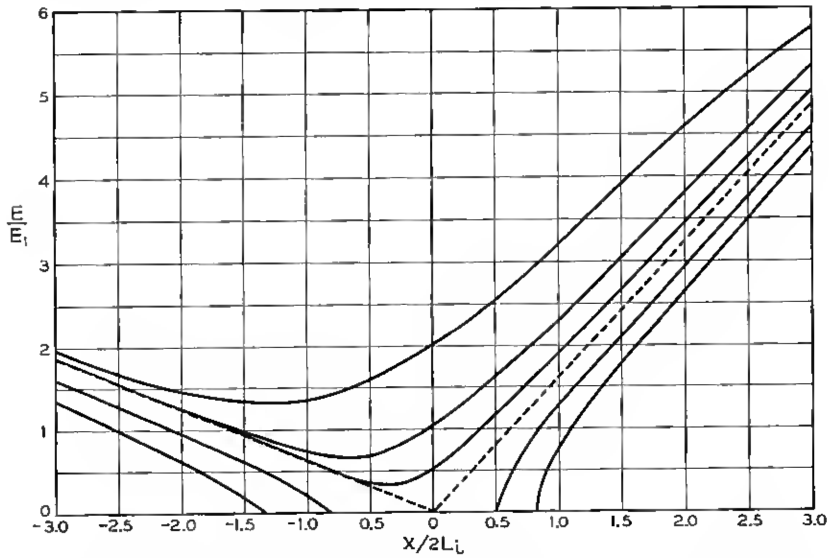

Fig. 9 — Field Distribution in the Range of Pure Drift for a fixed charge $N = N_i$, or $\alpha = 1$.

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APPENDIX A

Prim's Zero-Current Approximation

Prim's analysis is based on the assumption that the hole and electron currents are negligibly small differences between their drift and diffusion terms. Setting $J_p = J_n = 0$ then gives n and p as functions of the potential, which is found by substituting n and p into Poisson's equation and solving subject to the boundary conditions at the junctions. These conditions involve the applied bias and the majority carrier densities in the extrinsic regions. Since the current is assumed to vanish, the phenomena of carrier generation and recombination do not enter the problem and the results are independent of carrier mobility. The results will be exact when there is no applied voltage; the potential drop across the unit is then the built-in poteutial. In this appendix we use an internal consistency check to see for what values of applied bias the analysis

breaks down. First we find where the carrier concentration is in error by finding the bias at which the minimum drift current as calculated from $q\mu(n + p)E$ becomes equal to the total current, as found from the excess of generation over recombination in the intrinsic region. We then go on to find where the error in carrier concentration gives a sufficient error in space charge to affect the calculation of electric field. As we shall see, the zero-current approximation gives too low a carrier concentration in the interior of the intrinsic region. This will lead to serious errors in the field distribution only if the space charge of the carriers is important. When the bias is sufficiently high or the intrinsic region sufficiently narrow that the intrinsic region is swept so clean that the carrier space charge is, in fact, negligible, it will not matter that the calculated carrier density is too low, even by orders of magnitude. In such cases, the electric field is constant throughout most of the intrinsic region.

In the following we shall, for simplicity, take $b = 1$ and assume that the extrinsic regions are equally doped so that the problem is symmetrical.

Carrier Density

We now find where, on the zero current assumption, the drift current becomes equal to the total current. This involves knowing only the carrier concentrations and the field E_i in the center of the intrinsic region, where the drift current $q\mu(n + p)E_i$ is a minimum. By symmetry n and p are equal here and $n = p = n_i \exp(-qV_a/2kT)$ where V_a is the applied bias. The minimum field E_i is given by the total voltage drop V and the field penetration parameter η, which is the ratio of the minimum field to the average field. Thus $\eta = 2LE_i/V$ where $2L$ is the width of the intrinsic regions. The difference between V and V_a is the built-in voltage $(2kT/q)/\ell n(N/n_i)$ where N is the majority carrier concentration in the extrinsic regions. We now have for the drift current in the center of the intrinsic region

$$q\mu(n + p)E = q\eta D\left(\frac{qV}{kT}\right)\frac{n_i}{L}\exp\left(-\frac{qV_a}{2kT}\right) \tag{A1}$$

We next find the total current from the excess of generation over recombination in the intrinsic region. From the zero current assumption, $np = n_i^2 \exp(-qV_a/kT)$ is constant throughout the intrinsic region. Hence $g - r$ is constant. So the current $I = q(g - r)2L = qL(n_i^2 - np)/\tau n_i$ is

$$I = \frac{qLn_i}{\tau}\left[1 - \exp\left(-\frac{qV_a}{kT}\right)\right] \tag{A2}$$

Equating this to the drift current (A1) in the center of the intrinsic region gives

$$\left(\frac{L}{L_i}\right)^2 = \eta \frac{qV}{2kT} \operatorname{csch}\left(\frac{qV_a}{2kT}\right) \tag{A3}$$

The error in carrier concentration is less for narrower intrinsic regions and lower biases. Thus (A3) gives a curve of L versus V_a such that the zero current solution gives a good approximation to carrier concentration for points in the V_aL plane lying well below the curve. As expected, for zero bias, the solution is good for any value of L. However, for a bias of several kT/q, the solution for carrier concentration breaks down unless L is a very small fraction of a diffusion length.

Carrier Space Charge.

In Prim's analysis the carrier space charge is so low throughout most of the intrinsic region that the field remains approximately constant and equal to E_i. However there must be enough carriers present that the drift currents of holes and electrons can remove the carriers as fast as they are generated. In this section we ask where the space charge of the necessary carriers becomes large enough that its effect on the field can no longer be neglected. Let ΔE be the change in field due to the space charge in the intrinsic region (not counting the high field regions near the junctions). Unless ΔE is small compared to E_i the neglect of carrier space charge will not be justified. We shall find the ratio of ΔE to E_i.

If the field is to be approximately constant, then the hole and electron concentrations can easily be found from the hole and electrons currents. We shall deal with applied biases of at least a few kT/q, for which recombination is negligible and the total current $I = qg2L = qn_iL/\tau$. Since $g - r \doteq g$ is constant, the hole and electron currents are linear in x and, for constant field, are proportional to the hole and electron concentrations respectively. Thus the net space charge of the moving carriers $q(p - n)$ is proportional to x and varies from zero in the center of the intrinsic region to qp near the IP junction, where n is small compared to p and the current flows by hole drift, so $I = q\mu pE_i$. Thus the maximum charge is $I/\mu E_i$ and the total positive charge of the carriers on the P side of the center is $IL/2\mu E_i$. This gives a drop in field

$$\Delta E = \frac{aIL}{2q\mu E_i} = \frac{an_i}{2}\frac{kT}{qE_i}\frac{L^2}{L_i^2}$$

Dividing by $E_i = \eta V/2L$ gives

$$\frac{\Delta E}{E_i} = \frac{L^4}{\mathcal{L}^2 L_i^2}\left(\frac{kT}{\eta q V}\right)^2 \tag{A4}$$

Setting ΔE equal to some fraction, say 20 per cent of E_i gives a family of curves for V versus L with η as a parameter. Prim has plotted such curves in Fig. 11 of his paper. His curves will be good approximations when V for a given L and η lies above the V given by (A4).

Prim's results are expressed in terms of the parameters $U = qV/2kT$ and $\hat{L} = 2L/\mathcal{L}_e$ where $\mathcal{L}_e$ is the Debye length in the extrinsic material. $\mathcal{L}_e$ is given by the same formula as $\mathcal{L}$ except that N replaces n_i. Substituting these into (A4) and setting $\Delta E = E_i/5$ gives

$$\hat{L} = 3.57 \frac{NL_i}{n_i \mathcal{L}} \eta U \tag{A5}$$

Prim's U versus $\hat{L}$ curves will be accurate up to the point where they intersect the corresponding curves from (A5). For germanium a reasonable value of $NL_i/n_i\mathcal{L}$ is about 10^6. This says that Prim's curves go had at about $\hat{L} = 10^4$, which would he ahout 2.1×10^{-2} cm in germanium at 300°C.

Branching of the V versus L Curves

An effect which does not emerge from the zero-current analysis is that V may have several values for the same L and η. In other words tbe V versus L curve for given η will have more than one branch. Specifically, there will be a single V versus L curve up to a certain L at which the curve splits into three branches that diverge as L increases. This may be seen as follows: Consider an intrinsic region that is long compared to the diffusion lengtb. Suppose a bias is applied that is low enough not to appreciably affect the space charge and potential drop at the junctions. A current will flow and a proportional, ohmic voltage drop will be developed across the intrinsic region. If the intrinsic region is long enough, this ohmic voltage may become large compared to the built-in voltage before the voltage drop at the junctions has changed appreciably. In this range the field penetration parameter will he rising from zero to about unity as V increases from the built-in voltage and approaches the ohmic voltage. As the voltage continues to increase, the space charge begins to penetrate the intrinsic region and a majority of the voltage drop comes in the space charge regions. Let L be the effective length of current generation. When L is larger tban a diffusion

length but small compared to the length of the intrinsic region, then the voltage drop at the ends of the intrinsic region will be proportional to L^2 while the current, and consequently the minimum field, will be proportional to L. Thus η will be proportional to $1/L$ and will decrease as V increases and the region becomes more swept. Finally the two space charge regions meet; then η rises again with V and approaches unity. Hence, for a given η and length of intrinsic region, there will be three different values of V. For lower L the dip in the η versus V curve will be less, and there will be only one V for some values of η. Since η starts from zero at the built-in voltage and approaches unity for infinite voltage, there must be either one or three values of V for every η. Thus when the V versus L curve (or in Prim's notation the U versus $\hat{L}$ curve) branches, it branches at once into three curves. Prim's plot gives the upper branch in cases where all three are present.